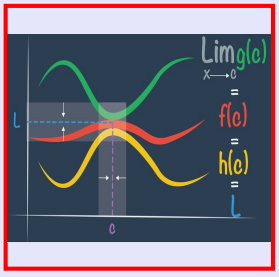
Calculus I

Lecture 19



Feb 19-8:47 AM

Given $f(x) = \frac{x-1}{x^2} = (x-1)x^{-2} = x^{-1} - x^{-2}$

1) Domain $x^2 = 0 \to x = 0$

$(-\infty, 0) \cup (0, \infty)$ $x = 0$ Vertical Asymptote

2) x-Int

$y = f(x) = 0 \to x - 1 = 0 \quad x = 1$

$\to (1, 0)$

3) Y-Int

$x = 0 \to 0$ is not in the domain

None

4) $f'(x)$ & $f''(x)$

$f'(x) = -x^{-2} + 2x^{-3}$

$f''(x) = 2x^{-3} - 6x^{-4}$

5) Find all points where $f'(x)$ & $f''(x)$ are Zero or undefined.

$f'(x) = -x^{-2} + 2x^{-3}$

$= x^{-3}(-x + 2)$

$= \frac{2-x}{x^3}$

$f'(x) = 0 \to 2 - x = 0 \to x = 2$

$(2, \frac{1}{4})$

$f'(x)$ is undefined $x^3 = 0 \to x = 0$

No point

$f''(x) = 2x^{-3} - 6x^{-4}$

$= 2x^{-4}(x - 3)$

$= \frac{2(x-3)}{x^4}$

$f''(x) = 0 \to 2(x-3) = 0 \to x = 3$

$(3, \frac{2}{9})$

$f''(x)$ is undefined $x^4 = 0 \to x = 0$

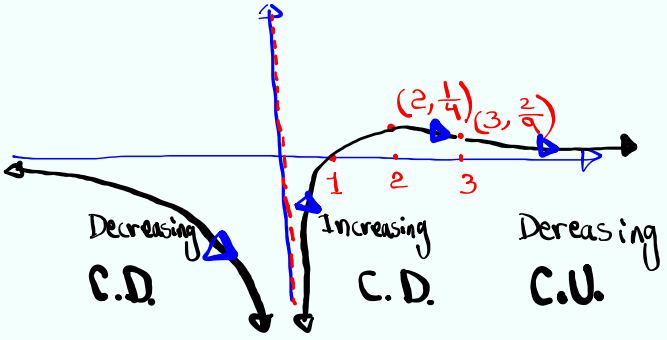
No point

Apr 28-8:45 AM

$f'(x) = \frac{2-x}{x^3}$ $\quad f''(x) = \frac{2(x-3)}{x^4}$

Points of interest $\quad x=0,\ x=2,\ x=3$

	-1		1		2.5		4	
x	$-\infty$		0		2	3		∞
$f'(x)$		−		+		−	−	
$f''(x)$		−		−		−		+



Apr 28-9:07 AM

Given $f(x) = 1 + 3x^2 - 2x^3$ $\quad f'(x) = 6x - 6x^2$ $\quad f''(x) = 6 - 12x$

1) Domain

Polynomial → $(-\infty, \infty)$

2) y-Int → $(0, 1)$

$f'(x) = 0 \quad 6x - 6x^2 = 0$

$6x(1-x) = 0$

$x = 0 \quad (0,1)$ $\qquad$ $x = 1 \quad (1,2)$

$f'(x)$ is never undefined.

3) $f'(x)$ & $f''(x)$

4) Find all points where $f'(x)$ & $f''(x)$ are Zero or undefined.

$f''(x) = 0$

$6 - 12x = 0$

$6(1 - 2x) = 0$

$x = \frac{1}{2}$

$\left(\frac{1}{2}, \frac{3}{2}\right)$

$f\left(\frac{1}{2}\right) = 1 + 3\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3$

$= 1 + \frac{3}{4} - \frac{1}{4}$

$= 1 + \frac{2}{4} = \frac{3}{2}$

$f''(x)$ is never undefined

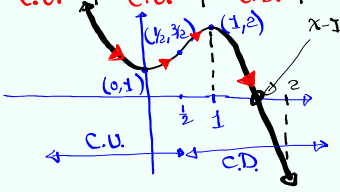
Apr 28-9:17 AM

$f'(x) = 6x - 6x^2$
$= 6x(1-x)$

$f''(x) = 6 - 12x$
$= 6(1-2x)$

Points of interest 0, 1/2, 1.

x	$-\infty$		0		$\frac{1}{2}$		1		∞
		-1		1		2			
$f'(x)$		−	0	+	+	0	−		
$f''(x)$		+		+	0	−		−	
$f(x)$		Decreasing C.U.		Increasing C.U.		increasing C.D.		Decreasing C.D.	



Show there is an x-Int between $x=1$ & $x=2$.

$f(x) = 1 + 3x^2 - 2x^3$ Polynomial → Cont. everywhere

$f(1) = 1 + 3(1)^2 - 2(1)^3 = 2$

$f(2) = 1 + 3(2)^2 - 2(2)^3 = -3$

By I.V.T., there is a place between 1 & 2 such that $f(x) = 0$

⇒ x-Int.

Apr 28-9:26 AM

$f(x) = x^4 - 2x^2 + 3$

1) Domain $(-\infty, \infty)$

$y = 0$ $f(x) = 0$

2) x-Ints $x^4 - 2x^2 + 3 = 0$

None $x^2 = \frac{-(-2) \pm \sqrt{4-12}}{2}$ ← No real Solutions

3) y-Int. $(0, 3)$

$f'(x) = 0$ $4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$x = 0$ $x = 1, x = -1$

$(0,3)$, $(1,2)$, $(-1,2)$

$f'(x)$ is never undefined.

$f\left(\frac{\sqrt{3}}{3}\right) = \left(\frac{\sqrt{3}}{3}\right)^4 - 2\left(\frac{\sqrt{3}}{3}\right)^2 + 3$

$= \frac{9}{81} - 2 \cdot \frac{3}{9} + 3$

$= \frac{9}{81} - \frac{54}{81} + 3$

$= \frac{9 - 54 + 243}{81}$

$= \frac{198}{81}$

4) Show $f(x)$ is an even function

$f(-x) = (-x)^4 - 2(-x)^2 + 3$

$= x^4 - 2x^2 + 3$

$= f(x)$ Symmetric w/ y-axis

5) $f'(x)$ & $f''(x)$

$f'(x) = 4x^3 - 4x$

$f''(x) = 12x^2 - 4$

6) Find all points where $f'(x)$ & $f''(x)$ are Zero or Undefined

$f''(x) = 0$

$12x^2 - 4 = 0$

$x^2 = \frac{1}{3}$ $x = \pm\sqrt{\frac{1}{3}}$

$\left(\frac{\sqrt{3}}{3}, \frac{198}{81}\right)$

$\left(-\frac{\sqrt{3}}{3}, \frac{198}{81}\right)$

$f''(x)$ is never undefined

Apr 28-9:41 AM

$f'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

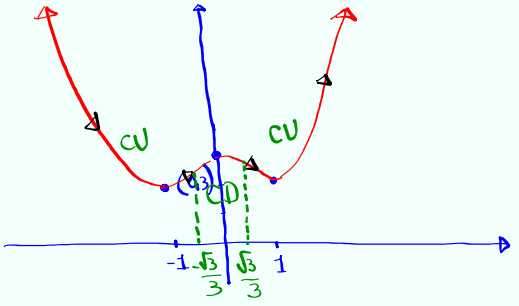
$= 4x(x+1)(x-1)$

$f''(x) = 12x^2 - 4$

$= 12\left(x^2 - \frac{1}{3}\right)$

$= 12\left(x + \frac{\sqrt{3}}{3}\right)\left(x - \frac{\sqrt{3}}{3}\right)$

x	$-\infty$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	∞
$f'(x)$	$-$	$+$	$+$	$-$	$-$	$+$	
$f''(x)$	$+$	$+$	$-$	$-$	$+$	$+$	
$f(x)$	Dec. C.U.	Inc. C.U.	Inc. C.D.	Dec CD	Dec CU	Inc. CU	

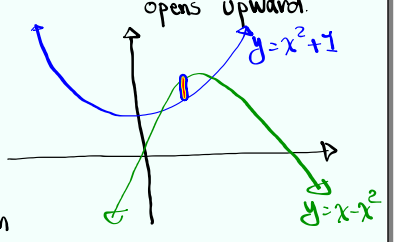


Apr 28-9:59 AM

What is the ~~maximum~~ Minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

$y = x - x^2$ opens downward

$y = x^2 + 1$ opens upward.



Vertical distance

$y_{\text{green graph}} - y_{\text{Blue graph}}$

$x - x^2 - (x^2 + 1) = x - x^2 - x^2 - 1 = -2x^2 + x - 1$

$f(x) = -2x^2 + x - 1$

$f'(x) = -4x + 1$ $\quad f'(x) = 0 \rightarrow x = \frac{1}{4}$

$f''(x) = -4$ $\quad f''(x) < 0$

Max at $x = \frac{1}{4}$

Max dist

$-2\left(\frac{1}{4}\right)^2 + \frac{1}{4} - 1 = -2 \cdot \frac{1}{16} + \frac{1}{4} - 1 = \frac{-1}{8} + \frac{1}{4} - 1 = \frac{1}{8} - 1$

$= \frac{-7}{8}$

Apr 28-10:14 AM

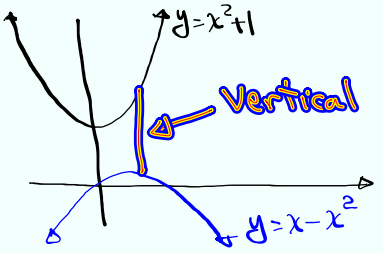
$x^2+1 = x - x^2$

$x^2+1 - x + x^2 = 0$

$2x^2 - x + 1 = 0$

$x = \frac{-(-1) \pm \sqrt{(-1)^2-4(2)(1)}}{2(2)} = \frac{1 \pm \sqrt{-7}}{4}$

No real solutions



$x^2+1 - (x-x^2)$

Top Bottom

$x^2+1-x+x^2$

$2x^2-x+1$

$f'(x) = 4x - 1 \rightarrow x = \frac{1}{4}$

$f''(x) = 4 \rightarrow f''(x) > 0$

C.U.

Minimum $\left(\frac{1}{4}, \frac{7}{8}\right)$

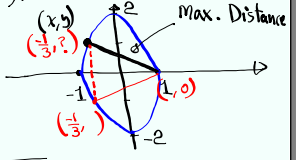
Apr 28-10:24 AM

Find a point or points on the graph of $x^2 + \frac{y^2}{4} = 1$ that are farthest away from the point $(1,0)$.

$y^2 = 4 - 4x^2$

Ellipse

Center $(0,0)$



Distance formula

$d = \sqrt{(x-1)^2 + (y-0)^2}$

$d = \sqrt{(x-1)^2 + y^2}$

$d = \sqrt{(x-1)^2 + 4-4x^2}$

$f(x) = (x-1)^2 + 4 - 4x^2$

$f'(x) = 2(x-1)^1 \cdot 1 + 0 - 8x = 2(x-1) - 8x = -6x - 2$

$f''(x) = -6 \rightarrow$ C.D.

Max

$f'(x) = 0$

$x = -\frac{1}{3}$

Points are $\left(-\frac{1}{3}, \pm\frac{4\sqrt{2}}{3}\right)$

$y^2 = 4 - 4x^2$

$= 4 - 4\left(-\frac{1}{3}\right)^2$

$= 4 - \frac{4}{9} = \frac{32}{9}$

$y^2 = \frac{32}{9}$

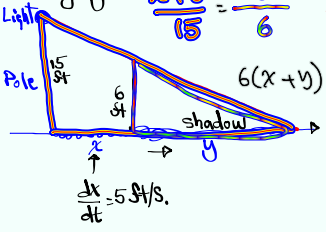
$y = \pm\frac{4\sqrt{2}}{3}$

Apr 28-10:31 AM

A street light is on top of pole that is 15-ft high.

A man that is 6ft tall walks away from the pole at the speed of 5 ft/s along a straight path.

How fast is the length of his shadow changing?



$\frac{x+y}{15} = \frac{y}{6}$ $\quad \frac{dy}{dt}$

$6(x+y) = 15y$

$6x = 9y$

$2x = 3y$

$2\frac{dx}{dt} = 3\frac{dy}{dt}$

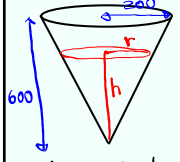
$2.5 = 3\frac{dy}{dt}$

$\frac{10}{3} = \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{10}{3}$ ft/s.

Apr 28-10:45 AM

Water is leaking out of an inverted conical tank.



$\frac{dr}{dt} = -$

$\frac{dh}{dt} = -$

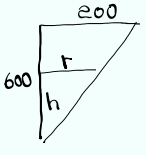
water is leaking at the rate of 10000 cm^3/min.

$\frac{dV}{dt} = -10000$

The tank has a height of 6 m and the radius of 2m.

600cm

200cm



$\frac{200}{600} = \frac{r}{h}$

$\frac{1}{3} = \frac{r}{h}$

$h = 3r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \cdot 3r$

$V = \pi r^3$

Find changes in the radius when radius is 1 m? what about its height?

$h = 3r$

$\frac{dh}{dt} = 3\frac{dr}{dt}$

$= 3 \cdot \frac{-1}{3\pi} = \boxed{\frac{-1}{\pi}}$ cm/min.

$\frac{dV}{dt} = \pi \cdot 3r^2 \frac{dr}{dt}$

$-10000 = 3\pi(100)^2 \frac{dr}{dt}$

$\frac{-10000}{3\pi(10000)} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-1}{3\pi}$ cm/min.

Apr 28-10:56 AM